Constructivist Learning and Teaching

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In reality, no one can *teach* mathematics. Effective teachers are those who can stimulate students to *learn* mathematics. Educational research offers compelling evidence that students learn mathematics well only when they *construct* their own mathematical understanding. (MSEB and National Research Council 1989, 58)

Radical changes have been advocated in recent reports on mathematics education, such as NCTM's *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics 1989) and *Everybody Counts* (MSEB and National Research Council 1989). Unfortunately, many educators are focusing on alterations in content rather than the reports' recommendations for fundamental changes in instructional practices. Many of these instructional changes can best be understood from a *constructivist* perspective. Although references to constructivist approaches are pervasive, practical descriptions of such approaches have not been readily accessible. Therefore, to promote dialogue about instructional change, each "Research into Practice" column this year* will illustrate how a constructivist approach to teaching might be taken for a specific topic in mathematics.

What Is Constructivism?

Most traditional mathematics instruction and curricula are based on the *transmission,* or *absorption,* view of teaching and learning. In this view, students passively "absorb" mathematical structures invented by others and recorded in texts or known by authoritative adults. Teaching consists of transmitting sets of established facts, skills, and concepts to students.

Constructivism offers a sharp contrast to this view. Its basic tenets—which are embraced to a greater or lesser extent by different proponents—are the following:

1. Knowledge is actively created or invented by the child, not passively received from the environment. This idea can be illustrated by the Piagetian position that mathematical ideas are *made* by children, not found like a pebble or accepted from others like a gift (Sinclair, in Steffe and Cobb 1988). For example, the idea "four" cannot be directly detected by a child's senses. It is a relation that the child superimposes on a set of objects. This relation is constructed by the child by reflecting on actions performed on numerous sets of objects, such as contrasting the counting of sets having four units with the counting of sets having three and five units. Although a teacher may have demonstrated and numerically labeled many sets of objects for the student, the mental entity "four" can be created only by the student's thought. In other words, students do not "discover" the way the world works like Columbus found a new continent. Rather they *invent* new ways of thinking about the world.
2. Children create new mathematical knowledge by reflecting on their physical and mental actions. Ideas are constructed or made meaningful when children integrate them into their existing structures of knowledge.
3. No one true reality exists, only individual interpretations of the world. These interpretations are shaped by experience and social interactions. Thus, learning mathematics should be thought of as a process of adapting to and organizing one's quantitative world, not discovering preexisting ideas imposed by others. (This tenet is perhaps the most controversial.)
4. Learning is a social process in which children grow into the intellectual life of those around them (Bruner 1986). Mathematical ideas and truths, both in use and in meaning, are cooperatively established by the members of a culture. Thus, the constructivist classroom is seen as a culture in which students are involved not only in discovery and invention but in a social discourse involving explanation, negotiation, sharing, and evaluation.

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5. When a teacher demands that students use set mathematical methods, the sense-making activity of students is seriously curtailed. Students tend to mimic the methods by rote so that they can appear to achieve the teacher's goals. Their beliefs about the nature of mathematics change from viewing mathematics as sense making to viewing it as learning set procedures that make little sense.

Two Major Goals

Although it has many different interpretations, taking a constructivist perspective appears to imply two major goals for mathematics instruction (Cobb 1988). First, students should develop mathematical structures that are more complex, abstract, and powerful than the ones they currently possess so that they are increasingly capable of solving a wide variety of meaningful problems.

Second, students should become autonomous and self-motivated in their mathematical activity. Such students believe that mathematics is a way of thinking about problems. They believe that they do not "get" mathematical knowledge from their teacher so much as from their own explorations, thinking, and participation in discussions. They see their responsibility in the mathematics classroom not so much as completing assigned tasks but as making sense of, and communicating about, mathematics. Such independent students have the sense of themselves as controlling and creating mathematics.

Teaching and Learning

Constructivist instruction, on the one hand, gives preeminent value to the development of students' personal mathematical ideas. Traditional instruction, on the other hand, values only established mathematical techniques and concepts. For example, even though many teachers consistently use concrete materials to introduce ideas, they use them only for an introduction; the goal is to get to the abstract, symbolic, established mathematics. Inadvertently, students' intuitive thinking about what is meaningful to them is devalued. They come to feel that their intuitive ideas and methods are not related to *real* mathematics. In contrast, in constructivist instruction, students are encouraged to use their own methods for solving problems. They are not asked to adopt someone else's thinking but encouraged to refine their own. Although the teacher presents tasks that promote the invention or adoption of more sophisticated techniques, all methods are valued and supported. Through interaction with mathematical tasks and other students, the student's own intuitive mathematical thinking gradually becomes more abstract and powerful.

Because the role of the constructivist teacher is to guide and support students' invention of viable mathematical ideas rather than transmit "correct" adult ways of doing mathematics, some see the constructivist approach as inefficient, free-for-all discovery. In fact, even in its least directive form, the guidance of the teacher is the feature that distinguishes constructivism from unguided discovery. The constructivist teacher, by offering appropriate tasks and opportunities for dialogue, guides the focus of students' attention, thus unobtrusively directing their learning (Bruner 1986).

Constructivist teachers must be able to pose tasks that bring about appropriate conceptual reorganizations in students. This approach requires knowledge of both the normal developmental sequence in which students learn specific mathematical ideas and the current individual structures of students in the class. Such teachers must also be skilled in structuring the intellectual and social climate of the classroom so that students discuss, reflect on, and make sense of these tasks.

An Invitation

Each article in this year's "Research into Practice" column will present specific examples of the constructivist approach in action. Each will describe how students think about particular mathematical ideas and how instructional environments can be structured to cause students to develop more powerful thinking about those ideas. We invite you to consider the approach and how it relates to your teaching—to try it in your classroom. Which tenets of constructivism might you accept? How might your teaching and classroom environment change if you accept that students must construct their own knowledge? Are the implications different for students of different ages? How do you deal with individual differences? Most important, what instructional methods are consistent with a constructivist view of learning?

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Constructivism and First-Grade Arithmetic

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For arithmetic instruction in the first grade, we advocate the use of games and situations in daily living in contrast to the traditional use of textbooks, workbooks, and worksheets. Our position is supported by the research and theory of Jean Piaget, called *constructivism,* as well as by classroom research (Kamii 1985, 1990).

Piaget's theory shows that children acquire number concepts by constructing them from the inside rather than by internalizing them from the outside. The best way to explain this statement is by describing children's reactions to one of the tasks Piaget developed with Inhelder (Inhelder and Piaget 1963).

The pupil is given one of two identical glasses, and the teacher takes the other one. After putting thirty to fifty chips (or beans, buttons, etc.) on the table, the teacher asks the pupil to drop a chip into his or her glass each time she drops one into hers. When about five chips have thus been dropped into each glass with one-to-one correspondence, the teacher says, "Let's stop now, and you watch what I am going to do." The teacher then drops one chip into her glass and says to the pupil, "Let's get going again." The teacher and the pupil drop about five more chips into each glass with one-to-one correspondence, until the teacher says, "Let's stop." The following is what has happened so far:

Teacher:

$$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$$

Pupil:

$$1 + 1 + 1 + 1 + 1 \qquad + 1 + 1 + 1 + 1 + 1$$

The teacher then asks, "Do we have the same amount, or do *you* have more, or do *I* have more?"

Four-year-olds usually reply that the two glasses have the same amount. When we go on to ask, "How do you know that we have the same amount?" the pupils explain, "Because I can see that we both have the same amount." (Some four-year-olds, however, reply that *they* have more, and when asked how they know that they have more, their usual answer is "Because.")

The teacher goes on to ask, "Do you remember how we dropped the chips?" and four-year-olds usually give all the empirical facts correctly, including the fact that only the teacher put an additional chip into her glass at one point. In other words, four-year-olds remember all the empirical facts correctly and base their judgment of equality on the empirical appearance of the two quantities.

By age five or six, however, most middle-class pupils deduce logically that the teacher has one more. When we ask these pupils how they know that the teacher has one more, they invoke exactly the same empirical facts as the four-year-olds.

No one teaches five- and six-year-olds to give correct answers to these questions. Yet children all over the world become able to give correct answers by constructing numerical relationships through their own natural ability to think. This construction from within can best be explained by reviewing the distinction Piaget made among three kinds of knowledge according to their sources—physical knowledge, logicomathematical knowledge, and social (conventional) knowledge.

Physical knowledge, on the one hand, is knowledge of objects in external reality. The color and weight of a chip are examples of physical properties that are *in* objects in external reality and can be known empirically by observation.

Logicomathematical knowledge, on the other hand, consists of *relationships* created by each individual. For instance, when we are presented with a red chip and a blue one and think that they are *different,* this difference is an example of logicomathematical knowledge. The chips are observable, but the difference between them is not. The difference exists neither *in* the red chip nor *in* the blue one, and if a person did not put the objects into this relationship, the difference would not exist for him or her. Other examples of relationships the individual can create between the chips are *similar, the same* in weight, and *two.*

Physical knowledge is thus empirical in nature because it has its source partly in objects. Logicomathematical knowledge, however, is not empirical knowledge, as its source is in each individual's head.

The ultimate sources of *social knowledge* are conventions worked out by people. Examples of social knowledge are the fact that Christmas comes on 25 December and that a tree is called "tree." Words such as *one, two,* and *three* and numerals such as 1, 2, and 3 belong to social knowledge, but the numerical concepts necessary to understand these numerals belong to logicomathematical knowledge.

Keeping the distinction among the three kinds of knowledge in mind, one can understand why most four-year-olds in the task described earlier said that the two glasses have the same amount. The four-year-olds had not yet constructed the logicomathematical relationship of number and could therefore gain only physical knowledge from the experience. From the appearance of the chips in the glasses, the pupils concluded that the amount was the same despite the fact that they remembered the way in which the chips had been dropped. Once the concept of number has developed, however, pupils will deduce from the same empirical facts that the teacher has one more chip regardless of the physical appearance.

New Goals for Beginning Arithmetic Instruction

If children develop mathematical understanding through their own natural ability to think, the goals of beginning arithmetic must be that pupils think and construct a network of numerical relationships. To add five and four, for example, pupils have to think $(1 + 1 + 1 + 1 + 1) + (1 + 1 + 1 + 1)$. This operation requires pupils to make two wholes (5 and 4) in their heads and then to make a higher-order whole (9) in which the original wholes (5 and 4) become parts. An example of a network of numerical relationships can be seen when pupils think about $5 + 4$ as one more than $4 + 4$ and as one less than $5 + 5$. Addition thus involves a great deal of thinking, that is, the making of relationships rather than mere skills (such as penmanship).

This definition of goals for instruction is very different from traditional instruction that focuses on correct answers and the writing of mathematical symbols. It is also very different from the assumption that pupils have to internalize "addition facts," store them, and retrieve them in computerlike fashion.

New Principles of Teaching

The following principles of teaching flow from constructivism and the preceding goals:

1. Encourage pupils to invent their own ways of adding and subtracting numbers rather than tell them how. For example, if pupils can play a board game with one die, we simply introduce a second die and let them figure out what to do.
2. Encourage pupils to exchange points of view rather than reinforce correct answers and correct wrong ones. For example, if a pupil says that six minus two equals three, we encourage pupils to agree or disagree with each other. Pupils *will* eventually agree on the truth if they debate long enough because, in logicomathematical knowledge, nothing is arbitrary.
3. Encourage pupils to think rather than to compute with paper and pencil. Written computation interferes with pupils' freedom to think and to remember sums and differences.

Classroom Activities

Paper-and-pencil exercises cause social isolation, mechanical repetition, and dependence on the teacher to know if an answer is correct. We, therefore, replace the textbook, workbook, and worksheets with two kinds of activities: games and situations in daily living.

Games, such as a modification of old maid in which pupils try to make a sum of ten with two cards, are well known to be effective. Although games are typically used only as a reward for pupils who have finished their work, we use games as a staple of instruction. Games give rise to compelling reasons for pupils to think and to agree or disagree with each other. When it is useful to know that $5 + 5$, $6 + 4$, $7 + 3$, and so on, all equal ten, pupils are much more likely to remember these combinations than when they write in workbooks to satisfy the teacher.

Situations in daily living also offer meaningful opportunities for pupils to construct mathematical relationships. Taking attendance, voting, collecting money, and sending notes home are examples of situations the teacher can use to encourage pupils to think. If four people brought their lunch, eight ordered the special, and six ordered soup and sandwich, the teacher can ask if everybody present has been accounted for. Pupils care about real-life situations and think much harder about these questions than about those in workbooks.

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